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ANALYSIS OF THE ORBIT OF ARIEL 1, 1962 - 15A, NEAR 15TH - ORDER RESONANCE

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SUMMARY

Ariel 1, the first international satellite, was launched on 26 April 1962, into an orbit inclined at 53.85° to the equator, with an initial perigee height near 390 km. On 8 May 1973 the orbit passed through 15th-order resonance and has been determined, with the RAE orbit refinement program PROP, at eight epochs between February and August 1973 using about 500 observations.

The orbital inclinations during the time of 15th-order resonance, as given by these eight orbits and 31 US Navy orbits, were fitted with a theoretical curve using the THROE computer program, the best fit giving $10^9\bar{C}_{15} = -370 \pm 14$ and $10^9\bar{S}_{15} = -114 \pm 31$.

The values of eccentricity were also successfully fitted using THROE and the results are discussed.

Departmental Reference: Space 462

CONTENTS

1 INTRODUCTION

Ariel 1, 1962-15A, the world's first international satellite, was launched on 26 April 1962 in the United States and carried experiments prepared by the United Kingdom. The satellite was cylindrical in shape with four paddles, had a length of 0.53 m, a diameter of 0.58 m and a mass of 60 kg [1]. The initial perigee and apogee heights were 389 km and 1214 km, and the inclination was 53.85°. The satellite is expected to decay in 1975.

The orbit of Ariel 1, which had an initial period of 100.9 minutes, has been contracting slowly under the action of drag and passed through 15th-order resonance in May 1973. A satellite orbit experiences 15th-order resonance when the orbital period is such that the Earth turns through 24°, relative to the orbital plane, between successive equator crossings, so that the ground track of the satellite over the Earth repeats after every 15 revolutions. The theory for such resonant orbits, developed by Allan[2-4], shows that changes occur in several orbital elements at the time of 15th-order resonance, and depend on the values of 15th-order harmonic coefficients in the geopotential.

In order to evaluate individual harmonic coefficients of order 15 and odd degree[5], the orbital changes at 15th-order resonance of a number of satellites at different inclinations need to be analysed. No results were previously available from a satellite orbit at inclination near 54°; so good results from Ariel 1 would be particularly useful.

Throughout its lifetime Ariel 1 has been a priority object for observing by the British optical and radar tracking stations, including the Hewitt cameras at Malvern and Edinburgh. In this Report eight sets of orbital elements have been determined over the resonance period, between February and August 1973, using the RAE orbit refinement program PROP[6], in the PROP 6 version, with Hewitt camera, optical and radar observations. These orbits, together with 31 US Navy orbits, have been analysed to determine equations for 15th-order harmonic coefficients in the geopotential.

At a future date the atmospheric rotation rate will be determined using (a) these eight sets of RAE elements, (b) further sets to be determined later, and (c) orbits obtained early in the satellite's life[7] using the Pegasus computer[8] and Minitrack observations.

2 THE OBSERVATIONS

The orbit was determined at eight epochs, between February 1973 and August 1973, from about 500 observations. A breakdown of the number of observations used on each is given in Table 1 overleaf.

Table 1

Sources of the observations used in each orbit determination

No.	Hewitt camera	Visual	Malvern radar	US Navy	Total
1	2	26		15	43
2		28		16	44
3				42	42
4	9	17		30	56
5	15	38		52	105
6			37	37	74
7			26	34	60
8	4	18		35	57

The observations came from four sources. The most accurate were the 30 observations on eight transits from the Hewitt camera at Malvern. These observations usually have an accuracy of 100 microseconds in time and 2 seconds of arc in position, and on average they improve the accuracy of the orbital inclination by a factor of about 5.

The observations in the second category were made by the volunteer visual observers reporting to the Science Research Council, Appleton Laboratory, Slough. These visual observations usually have accuracies between 1 and 3 minutes of arc.

The largest group consisted of US Navy observations kindly supplied by the US Naval Research Laboratory. These observations had a topocentric accuracy of about 2 minutes of arc. The remaining observations were made by the radar tracker at RRE, Malvern, with a directional accuracy of about 3 minutes of arc and range accuracy of about ½ km.

3 THE ORBITS OBTAINED

The eight sets of orbital elements obtained are listed in Table 2 on page 5, with the standard deviation shown below each value. As usual with PROP, the epoch is at 0 hours on the day of the determination and the definitions of the symbols used are given under Table 2.

In the PROP model[6] the mean anomaly, M, is represented by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + \dots , \qquad (1)$$

Table 2

Orbital parameters of Ariel 1, with standard deviations, at the 8 epochs

No.	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	ε	N	D
1*	41723	1973 Feb 10	6889.3972 14	0.02262 4	53.8328 6	324.6305 49	201.47 7	297.07 7	5465.4698 16	0.0566 1	-0.00075 9	-	0.78	37	9.0
2	41778	1973 Apr 6	6883.1872 9	0.02220 2	53.8328 9	77.1924 37	2.47 4	108.83 4	5472.8677 11	0.1190 2	0.00052 9	-	0.52	39	8.8
3	41820	1973 May 18	6876.0951 8	0.02227 2	53.8139 16	247.4798 23	117.48 4	126.19 4	5481.3352 10	0.0675 4	0.00042 8	-	0.52	37	9.9
4*	41832	1973 May 30	6874.7627 6	0.02170 2	53.8038 1	193.1654 5	150.70 2	33.28 2	5482.9277 8	0.0487 4	-0.00208 10	-	0.56	52	6.7
5*	41839	1973 Jun 6	6874.3016 4	0.02130 1	53.7971 0	161.4469 1	170.07 1	256.35 1	5483.4787 4	0.0282 2	-0.01206 42	0.00417 19	0.92	78	4.0
6	41860	1973 Jun 27	6873.1606 4	0.02019 2	53.7925 13	66.2716 24	230.92 5	222.31 5	5484.8438 5	0.0314 3	-	-	0.64	66	6.6
7	41874	1973 Jul 11	6872.4620 8	0.01992 1	53.7948 13	2.7900 19	273.23 6	334.07 6	5485.6803 10	0.0241 6	-	-	0.59	46	4.5
8*	41902	1973 Aug 8	6870.8599 5	0.02079 1	53.7950 1	235.7746 12	356.16 1	233.74 1	5487.5991 6	0.0517 1	0.00063 5	-	0.39	46	7.9

* Orbits using Hewitt camera observations

Key: MJD = modified Julian day
a = semi major axis (km)
e = eccentricity
i = inclination (deg)
Ω = right ascension of ascending node (deg)
ω = argument of perigee (deg)
M_0 = mean anomaly at epoch (deg)
M_1 = mean motion n (deg/day)
M_2, M_3, M_4 = later coefficients in the polynomial for M
ε = measure of fit
N = number of observations used
D = time covered by the observations (days)

where t is the time measured from epoch, and a suitable number of M-coefficients can only be obtained by trial and error, as the fitting depends on the irregularity of the drag. For five of the eight orbits the best results were obtained using M_0 to M_3 ; two orbits required only M_0 to M_2 ; and one, No.5, required M_0 to M_4 . The orbits fitted the observations well, with ϵ , the quantity indicating the measure of fit, ranging between 0.39 and 0.92.

The accuracies of the inclination, i , and the right ascension of the node, Ω , are rather varied. The best of the four orbits with Hewitt camera observations, No.5, with 15 observations on four transits, has a standard deviation of 0.00001° in inclination, corresponding to 2m across-track; two others have standard deviations of 0.0001° and the fourth 0.0006°. These four orbits with Hewitt camera observations give better results for the inclination, with average sd of 0.0002°, than the other four orbits, whose average sd is 0.0013°. For the right ascension of the node, Ω , the orbits without Hewitt camera observations have standard deviations near 0.003°, while three of the four Hewitt camera orbits have standard deviations of 0.0012° or less (though the fourth has the surprisingly large sd of 0.0049°).

For the other orbital parameters the accuracies are more uniform. The average sd in the semi major axis, a , is about 1 m, and the average sd for the eccentricity, e , is 0.00002. The standard deviations in argument of perigee, ω , and mean anomaly, M_0 , are equal and about 0.04° on average; finally, the mean motion, M_1 , is determined with an average accuracy of 1 part in 5 million. These last five parameters - a, e, ω, M_0 and M_1 - are not greatly improved by the inclusion of Hewitt camera observations.

4 THE ACCURACY OF THE OBSERVATIONS

The residuals of the observations have been obtained using the ORES computer program[9] and sent to the observers. The accuracies of selected observing stations, with ten or more observations accepted in the orbit determinations, are listed in Table 1, page 7, along with the number of accepted and rejected observations. The observations from the US Navy station 29 are geocentric, and if they were given in the same form as the other (topocentric) observations, their angular rms residuals would increase by a factor of about 5.

Table 3

Residuals for selected observing stations

Station	Number of observations		Rms residuals				abc residuals minutes of arc	
			Range km	Minutes of arc				
	Total	Rejected		RA	Dec	Total	RA	Dec
1 US Navy	22	3		2.3	3.1	3.9	-	-
2 US Navy	12	2		3.3	4.0	5.2	-	-
3 US Navy	21	4		2.8	2.9	4.0	-	-
4 US Navy	16	4		1.9	2.3	2.9	-	-
5 US Navy	17	1		2.4	1.7	2.9	-	-
6 US Navy	11	0		2.0	1.9	2.7	-	-
29 US Navy	162	9	0.4*	0.6*	0.6*		-	-
2303 Malvern Hewitt camera	30	0		0.07	0.02	0.07	-	-
2304 Malvern radar	63	15	0.6	4.2	4.8		-	-
2414 Bournemouth	11	1		8.4	4.8	9.7	5.5	4.0
2419 Tremadoc	14	0		5.2	3.3	6.2	2.3	2.0
2420 Willowbrae	16	2		4.9	1.6	5.1	3.6	0.9
2421 Malvern 4	51	5		3.4	2.7	4.4	1.5	1.4

* geocentric

The abc residuals (average in best conditions), defined as the arithmetic mean of the best 70-80% of the residuals, are indicated in Table 3, for the four visual observers who make their observations relative to the star background. The abc residual is included because visual observers are encouraged to make observations on every possible occasion, even when few stars are visible and accurate observation is difficult.

5 ANALYSIS OF THE VARIATION IN INCLINATION

The eight values of inclination from the PROP orbits in Table 2, with their sd, and the 31 values of inclination from the US Navy orbits are plotted in Fig.1. These values clearly reveal a large variation in inclination centred on MJD 41810, the date of 15th-order resonance.

Fig.2 shows the 39 values of inclination after removal of lunisolar and zonal harmonic perturbations and the effects of an atmospheric rotation rate, Λ, of 1.1 rev/day, and, for the orbits of Table 2, the $J_{2,2}$ perturbations. The methods of calculation used in removing these perturbations are described in Ref.10. A further correction of -0.004° was made to the US Navy orbits to remove an apparent bias. Such a bias, which has also been noted with 1968-86A[11] and 1970-111A[10], could arise from the use of different sets of values for the

even zonal harmonics in the RAE and US Navy orbit determination programs, because $\dot{\Omega}$ is computed from the theoretical expression involving mainly the even harmonics and cos i : thus if $\dot{\Omega}$ is slightly in error, due to errors in the assumed values of the harmonics, the orbit determination program achieves its best fit by altering cos i slightly to compensate - the resulting improvement in fitting Ω outweighs the deterioration in i, especially if there are no observations near apex.

The change in the inclination at times near resonance due to 15th-order harmonics in the geopotential may be written in the form

$$\begin{aligned}\frac{di}{dt} = nG\left(\frac{R}{a}\right)^{15}\Big[&\bar{C}_{15}\sin\Phi - \bar{S}_{15}\cos\Phi \\ &+ \bar{C}_{30}\sin 2\Phi - \bar{S}_{30}\cos 2\Phi + \text{terms in } (\bar{C}_{45},\bar{S}_{45}) \begin{matrix}\cos\\ \sin\end{matrix}\, 3\Phi\text{, etc.} \\ &+ e\Big\{A_i\sin(\Phi-\omega) - B_i\cos(\Phi-\omega) + C_i\sin(\Phi+\omega) - D_i\cos(\Phi+\omega) \\ &+ \text{terms in } \begin{matrix}\cos\\ \sin\end{matrix}\,(k\Phi - q\omega)\Big\}\Big]\,,\end{aligned} \qquad (2)$$

where k and q are integers $\geqslant 1$,

$\bar{C}_{15}$, $\bar{S}_{15}$, A_i, B_i etc., are constants related to geopotential coefficients,

$G = 0.5877\,(15 - \cos i)(1 + \cos i)\sin^{13} i$,

n is the mean motion ($= \dot{M}$), and

R is the Earth's equatorial radius (6378.1 km).

The 'resonance angle' Φ is given by

$$\Phi = \omega + M + 15(\Omega - \nu)\,,$$

where ν is the sidereal angle. The terms written explicitly in equation (2) correspond to $(k,q) = (1,0), (2,0), (3,0), (1,1), (1,-1)$: other terms that may be significant are $(k,q) = (1,\pm2), (1,\pm3), \ldots$ and $(2,\pm1), (2,\pm2), \ldots$.

The THROE computer program developed by Gooding[12] provides a least-squares fitting of equation (2), numerically integrated, to the observed variation in inclination near 15th-order resonance, after lunisolar and other relevant perturbations have been removed. In the fitting the US Navy values were given sd of 0.003°, and the values from Table 2 were given their quoted sd, except for orbits 4, 5 and 8, for which the sd was increased to 0.0005° because Earth-tide perturbations (probably of order 0.0003°) had not been taken into account.

THROE has been improved since the version listed in Ref.12, so that specified pairs of k and q values can be selected. Several runs were computed with different pairs: the first, run 1, had the $\begin{matrix}\sin\\ \cos\end{matrix}\,\Phi$ terms and the

main e terms, i.e. those corresponding to (k,q) = (1,0), (1,1) and (1,-1). The values obtained for the corresponding constants ($\bar{C}_{15}$,$\bar{S}_{15}$), (A_i,B_i) and (C_i,D_i) are given in Table 4 (first column). The fit is good, with ε near 1, as it should be*, but C_i and D_i are indeterminate. So run 2 was computed with just (k,q) = (1,0) and (1,1), giving the values in column 2 of Table 4.

Table 4

Values of $\bar{C}_{15}$, $\bar{S}_{15}$ etc., from various fittings of the inclination

Run	1	2	3	4	5
$10^9\ \bar{C}_{15}$ (1,0)	-357 ± 25	-370 ± 14	-379 ± 33	-354 ± 26	-402 ± 19
$10^9\ \bar{S}_{15}$ (1,0)	-108 ± 34	-114 ± 31	-127 ± 38	-122 ± 36	-186 ± 35
$10^9\ \bar{C}_{30}$ (2,0)			-55 ± 47		
$10^9\ \bar{S}_{30}$ (2,0)			30 ± 53		
$10^9\ A_i$ (1,1)	4163 ± 1555	3607 ± 1331	3520 ± 1386	2600 ± 1815	1179 ± 1398
$10^9\ B_i$ (1,1)	2994 ± 468	2885 ± 433	2918 ± 460	3135 ± 1215	2529 ± 399
$10^9\ C_i$ (1,-1)	-1142 ± 1667				
$10^9\ D_i$ (1,-1)	- 197 ± 821				
Other terms				(1,2)	(1,-2)
ε	0.84	0.82	0.83	0.83	0.73

The fit is slightly better - ε decreases from 0.84 to 0.82 - and the sd are lower; so this run is quite satisfactory. In run 3 the effect of adding the main 30th-order term (2,0) was tested: there was no improvement in fit, and $\bar{C}_{30}$ and $\bar{S}_{30}$ were indeterminate, as Table 4 shows. The terms (1,2) and (1,-2) were then added, first together and then separately. With both added together the sd increased considerably, but the increase in sd was less when either the (1,2) or the (1,-2) terms were added separately, as shown in runs 4 and 5 of Table 4. (The numerical values of the (1,2) and (1,-2) coefficients are not given because their definition is lengthy and their values were not well-determined.) Finally a linear variation in di/dt was added to take

* ε^2 is the sum of the squares of the weighted residuals, divided by the number of degrees of freedom.

account of any error in the correction for atmospheric rotation. However the value was not significant, $(-1.3 \pm 10.2) \times 10^{-6}$ deg/day , and the values of $\bar{C}_{15}$ and $\bar{S}_{15}$ were the same as for run 2.

The five values of $\bar{C}_{15}$ in Table 4 are satisfactorily consistent: they do not differ from one another by much more than the sum of their standard deviations. The same is true for $\bar{S}_{15}$. So we may conclude that the values obtained are not too sensitive to the model used for the fitting. Of the various models tried, run 2 seems the best, having the smallest number of parameters, the lowest sd and a good-looking fit, shown in Fig.2.

The values of $\bar{C}_{15}$ and $\bar{S}_{15}$ can be expressed as linear functions of the individual normalised 15th-order tesseral harmonic coefficients $\bar{C}_{\ell,15}$ and $\bar{S}_{\ell,15}$ of odd degree (ℓ = 15, 17, 19 ...). The values of $\bar{C}_{15}$ and $\bar{S}_{15}$ obtained here will supersede the preliminary values used in Ref.5, together with results for other satellites, to determine harmonic coefficients of order 15 and odd degree up to degree 31.

6 ANALYSIS OF THE VARIATION IN ECCENTRICITY

Theory indicates that the variation of e should take the form[13]

$$\frac{de}{dt} = nG\left(\frac{R}{a}\right)^{15}\left[A \sin(\Phi - \omega) - B\cos(\Phi - \omega) + C\sin(\Phi + \omega) - D\cos(\Phi + \omega) + \text{terms in}\left\{e^{|q|-1}(q - \tfrac{1}{2}ke^2)\begin{matrix}\cos\\ \sin\end{matrix}(k\Phi - q\omega)\right\}\right]$$

$$+ \text{ terms due to air drag, zonal harmonic and lunisolar perturbations.} \qquad (3)$$

Again k and q are integers and the main terms, given explicitly, are those with $(k,q) = (1,\pm 1)$. The subsidiary terms which are most likely to be significant[13] are those with $(k,q) = (2,\pm 1)$ and $(1,\pm 2)$. The THROE computer program can be used for fitting e in the same way as for i .

The change in $\dot{e}$ due to drag, $\dot{e}_{drag}$, is very important for Ariel 1, and may usefully be split into two components: (a) the decrease caused by the decrease in semi major axis a , assuming the perigee height is constant; and (b) a small increase due to the small decrease in perigee height. Of these, (a) is purely geometrical and given exactly by $\dot{a}(1 - e)/a$ or, in terms of M_1 and M_2 , $-4M_2(1 - e)/3M_1$; but (b) depends on the scale height and oblateness of the upper atmosphere, being given by equation (2) of Ref.14 as

$$\frac{2H_1M_2}{3aeM_1}\left(1 - 2e + \frac{H_1}{4ae} - \frac{2\varepsilon'}{e}\sin^2 i \cos 2\omega\right) \qquad (4)$$

for $ae > 3H_1$, where H is the density scale height, H_p the value of H at perigee and H_1 the value at a height $1.5H_p$ above perigee, and ε' is the ellipticity of the atmosphere. The PROP model, which is incorporated in THROE, does not include the ε' term and takes a constant value of H_1 , which has to be specified, over the whole time-interval. There are thus three possible sources of error in the $\dot{e}_{drag}$ term: (1) the neglect of the ε' term, which is about 15% of the main term; (2) errors in the choice of the average value of H_1 , possibly amounting to about 10%; and (3) the effect of variations in H_1 , which varies from week to week, and even from day to day, in response to the variations in solar ultraviolet radiation and the influx of solar particles. Two of these three possible deficiencies in the calculation of $\dot{e}_{drag}$ within THROE can be covered by including, in the fitting of e , either a linear variation with time (i.e. a constant term in de/dt) or a term in $\cos 2\omega$, i.e. taking $(k,q) = (0,2)$. It is not possible to take account of the variations of H with time, which remain a likely source of error.

The zonal harmonic perturbations are allowed for within THROE; the luni-solar perturbations were calculated, using PROD[15], but were found to be negligible, < 0.00002 .

THROE was run first with $H = 60$ km and different accuracies for the US Navy values. The results are given in Table 5 (page 12), runs 1-4. For run 1 just the main terms were evaluated with $(k,q) = (1,\pm1)$ and the usual accuracy estimate of 0.00004 given to the US Navy values of e . Run 2 has a linear variation of e with time, Lt say, to compensate for any deficiencies in the calculation of $\dot{e}_{drag}$ within THROE. This gives a better fit, with ε , the measure of fit, decreasing from 2.27 to 1.87. Runs 3 and 4 are a repeat of 1 and 2 with the US Navy accuracy changed to 0.00008, so that more weight is given to the more accurate PROP values. This improved the fit on both runs, with the solution containing the Lt term still the better. The subsidiary terms most likely to be significant were then added in turn, $(k,q) = (2,\pm1)$, $(1,\pm2)$ and $(1,0)$. In each of the three runs the extra terms were not well-determined - only one of the ten values obtained had a standard deviation of less than half its value - and the standard deviations on the main terms increased. These three runs are not included in Table 5. Runs 5 and 6 are repeats of runs 3 and 4 with $H = 50$ km, a more realistic value of H , as the orbit was determined at a time of fairly low solar activity. The fit was improved, as shown by the lower values of ε . On comparing these pairs of runs, 3 with 4 and 5 with 6, it can be seen that the linear term L makes a great deal of difference and the correlation matrix (not given here) shows that

Table 5

THROE fittings to the values of eccentricity

Run	H km	US Navy accuracy	Main terms with $(k,q) = (1,\pm1)$				Linear term 10^6L	ε
			10^9A	10^9B	10^9C	10^9D		
1	60	0.00004	16 ± 42	-21 ± 21	36 ± 76	-194 ± 35		2.27
2	60	0.00004	75 ± 37	-183 ± 43	-46 ± 65	-103 ± 36	-2.5 ± 0.6	1.87
3	60	0.00008	57 ± 33	-15 ± 16	-42 ± 63	-146 ± 30		1.53
4	60	0.00008	107 ± 25	-214 ± 37	-44 ± 48	-42 ± 28	-3.2 ± 0.6	1.10
5	50	0.00008	67 ± 30	-42 ± 14	31 ± 57	-106 ± 27		1.38
6	50	0.00008	108 ± 25	-203 ± 36	-38 ± 47	-21 ± 28	-2.6 ± 0.6	1.09
7	50	0.00008	-119 ± 29	-62 ± 14	60 ± 56	-190 ± 27		1.36
8	50	0.00008	-76 ± 24	-229 ± 35	-12 ± 45	-102 ± 27	-2.7 ± 0.5	1.04
9	50	0.00004	-131 ± 29	-58 ± 14	17 ± 54	-238 ± 24		1.57
10	50	0.00004	-93 ± 27	-161 ± 32	-26 ± 49	-180 ± 27	-1.6 ± 0.4	1.35

there is a high correlation (0.95) between L and B, the second of the main terms.

At this stage of the analysis an extra run with the PROP values deliberately degraded in accuracy indicated that there might be systematic differences, dependent on $\sin\omega$, between the US Navy and PROP values. This could arise from an incorrect restoration of the odd harmonic perturbation to e, which is removed from the raw US Navy orbits. In an attempt to eliminate this bias, runs 5 and 6 were repeated with $0.0001 \sin\omega$ subtracted from each of the eight PROP values of e, runs 7 and 8. (It would have been more logical, but more laborious to add $0.0001 \sin\omega$ to the 31 US Navy values of e: the effect would be similar.)

After the effect of incorrect restoration of the odd harmonic perturbation to the US Navy values had been removed, the normal accuracy estimate of 0.00004 could be used for these orbits. The resulting values are shown as runs 9 and 10 in Table 5. The accuracy for the PROP value of e at MJD 41778 was relaxed for these two runs and a revised version of the THROE program used, which included the most recent values for the zonal harmonics.

Two further runs were computed but not recorded in Table 5, including the term $(k,q) = (0,2)$. Adding this term would compensate for any deficiency in

THROE due to the neglect of the ε' term in equation (4). However, the four main terms were not appreciably different from those in runs 9 and 10; the linear term had the same value as in run 10, perhaps indicating that the main error in the $\dot{e}_{drag}$ term, for this satellite, comes from the variation of H_1 rather than from the neglect of the ε' term; and the extra coefficients were not well-determined. Therefore runs 9 and 10 were considered to provide the best fit.

The two sets of values for the main terms given in runs 9 and 10 at a first glance seem to be markedly different. On closer inspection the corresponding values in each run are seen to agree to within 1.2 times the sum of their sd, except for B. The values for B would not be expected to agree, as the linear term is so highly correlated with B. The curves given by runs 9 and 10 have been plotted in Fig.3 together with the values of e being fitted. Both curves fit the values of e fairly well: the unbroken curve representing run 10 is probably to be preferred because it has a lower ε and gives a better fit to the US Navy values between MJD 41779 and 41800.

In equation (3), the constants A, B, C and D are related to the geopotential coefficients of order 15 and degree 16, 18, ... Expressions for A, B, C and D in terms of $\bar{C}_{16,15}$, $\bar{C}_{18,15}$ etc and $\bar{S}_{16,15}$, $\bar{S}_{18,15}$ etc. are available from equations (14)-(16) of Ref.10.

The constants A, B, C and D of equation (3) are related theoretically to the A_i, B_i, C_i and D_i of equation (2). On comparing equations (55) and (59) of Ref.4, we have

$$\left.\begin{aligned} A_i, B_i &= -15 \operatorname{cosec} i(A,B) = -18.6(A,B) \\ C_i, D_i &= (15 \operatorname{cosec} i - 2 \cot i)(C,D) = 17.1(C,D) \end{aligned}\right\} \tag{5}$$

where the numerical values are for $i = 53.8^\circ$. Substituting the values of A, B, C and D from Table 5, run 10, into equations (5) gives

$$\left.\begin{aligned} 10^9 A_i &= 1730 \pm 500 & 10^9 B_i &= 2990 \pm 600 \\ 10^9 C_i &= -440 \pm 840 & 10^9 D_i &= -3080 \pm 460 \end{aligned}\right\} . \tag{6}$$

Approximate values of A_i and B_i, found as a by-product when analysing the inclination, have been given in Table 4: it is most satisfactory that all ten values of A_i and B_i in Table 4 agree with the values in equations (6) to within little more than the sum of the standard deviations. The values of C_i and D_i in Table 4 were indeterminate.

The value of B from run 9 in Table 5 gives $10^9 B_i = 1080 \pm 260$, which disagrees with the values in Table 4, thus tending to confirm that run 10 may be preferable to run 9.

7 CONCLUSIONS

Eight sets of orbital elements were determined near 15th-order resonance, which occurred on 8 May 1973, using the PROP computer program. The orbits have mean standard deviations of 0.001° in inclination and 0.00002 in eccentricity.

The eight values of inclination from the PROP orbits together with 31 US Navy values were fitted with a theoretical curve using THROE. The results of various fittings are given in Table 4, with run 2 giving the best results, plotted in Fig.2. These results will be used in a future redetermination of the harmonic coefficients of order 15 and odd degree. (The values used in Ref.5 were preliminary values.)

Fitting to the values of eccentricity was somewhat experimental, and many different fittings were tried. Ten of these are recorded in Table 5, and run 10, shown in Fig.3, appears to be the best.

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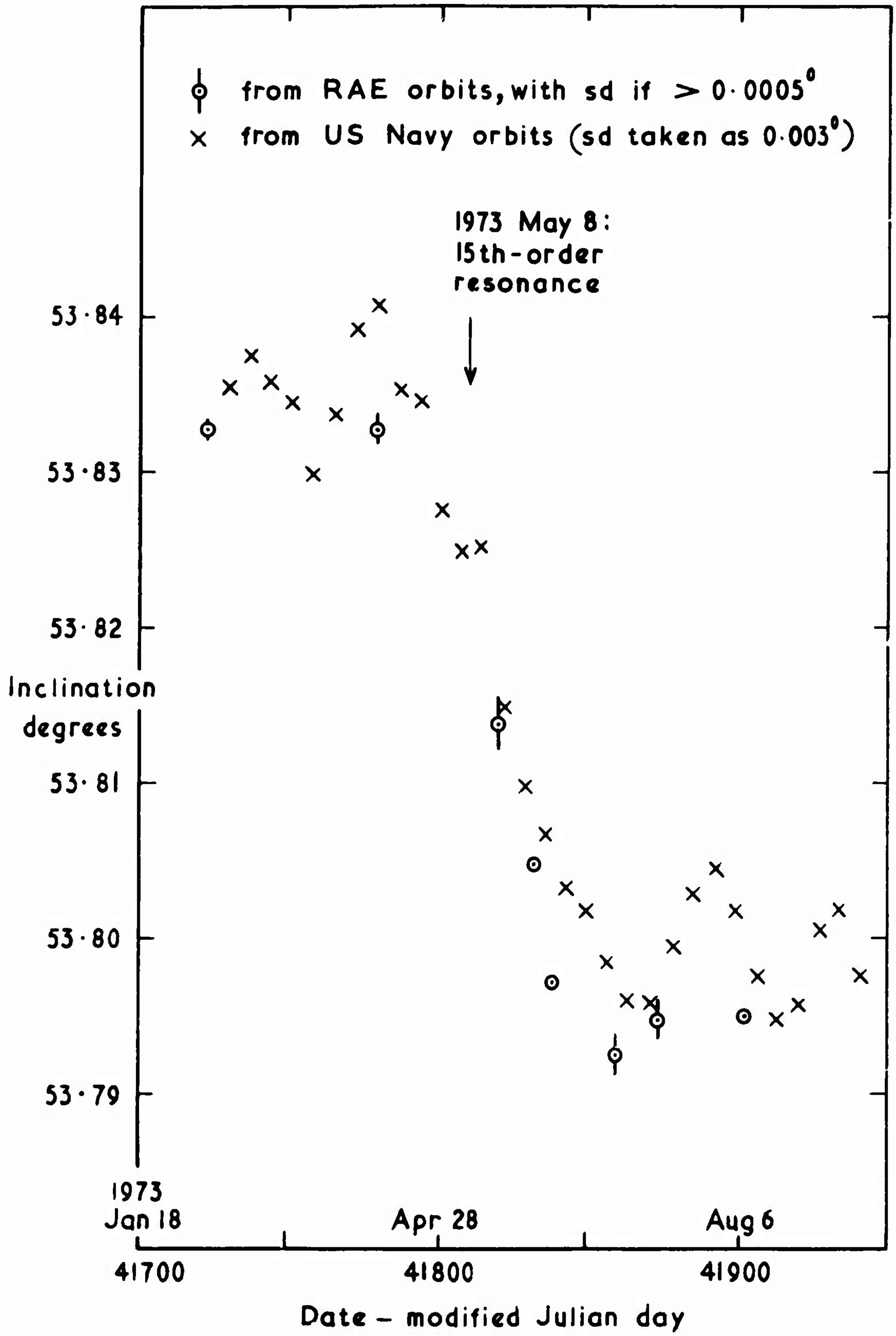

Fig 1 Values of inclination from 8 RAE orbits and 31 US Navy orbits

Fig. 2

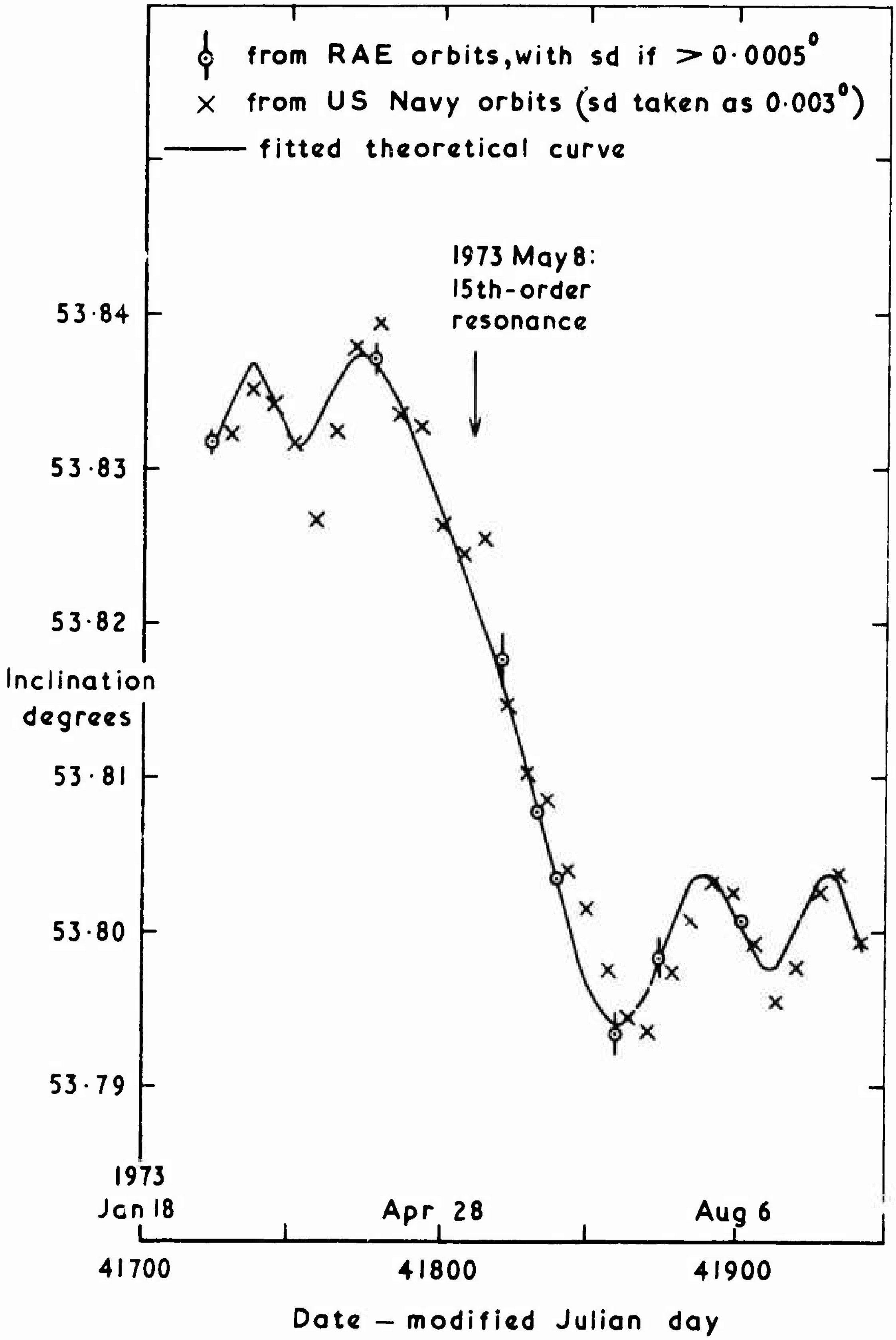

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Fig. 2 Values of inclination near 15th-order resonance, corrected for atmospheric rotation ($\Lambda = 1 \cdot 1$), zonal harmonic, lunisolar, and $J_{2,2}$ perturbations, with fitted theoretical curve

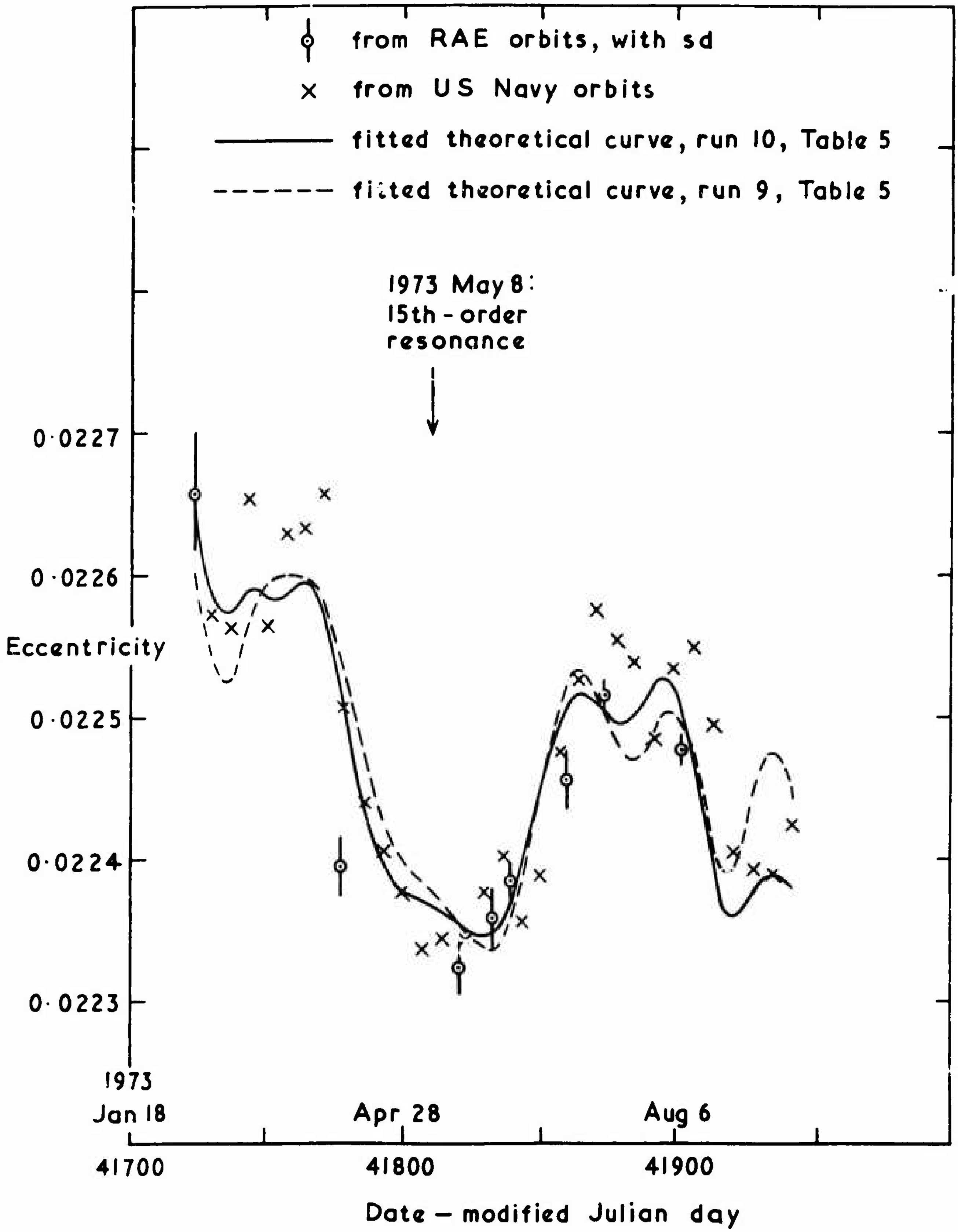

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Fig.3 Values of e from RAE and US Navy orbits, with fitted curves